



Diversification, the Capital Asset Pricing Model, and the Cost of Equity Capital

Risk as Variability in Return

The rate of return an investor receives from holding a stock for a given period of time is equal to the dividends received plus the capital gains in the period divided by the initial market value of the security:

$$R = \frac{\text{Dividends} + (\text{Ending price} - \text{Beginning price})}{\text{Beginning price}}$$

Alternatively, return can be viewed as the dividend yield plus the percentage capital appreciation:

$$R = \text{Dividend yield} + \text{Percentage capital appreciation}$$

Suppose an investor buys one common share of Du Pont for \$100 on January 1. Over the year he or she receives \$4 in dividends and sells the share for \$108 on December 31. The return on this investment is 12%:

$$R_{\text{Du Pont}} = \frac{\$4 + (\$108 - \$100)}{\$100} = \frac{\$12}{\$100} = .12$$

or

$$R_{\text{Du Pont}} = 4\% \text{ dividend yield} + 8\% \text{ appreciation} = 12\%$$

If the ending price is \$85, the return is -11%.

The return on any security can be viewed as the cash the security holder receives (including liquidation at the end of the period) divided by the initial investment. Investing in a savings account that offers a 5% interest rate results in an annual return of 5%:

$$R_{\text{Savings account}} = \frac{\$5 + (\$100 - \$100)}{\$100} = .05$$

This case was prepared as the basis for class discussion rather than to illustrate either effective or ineffective handling of an administrative situation.

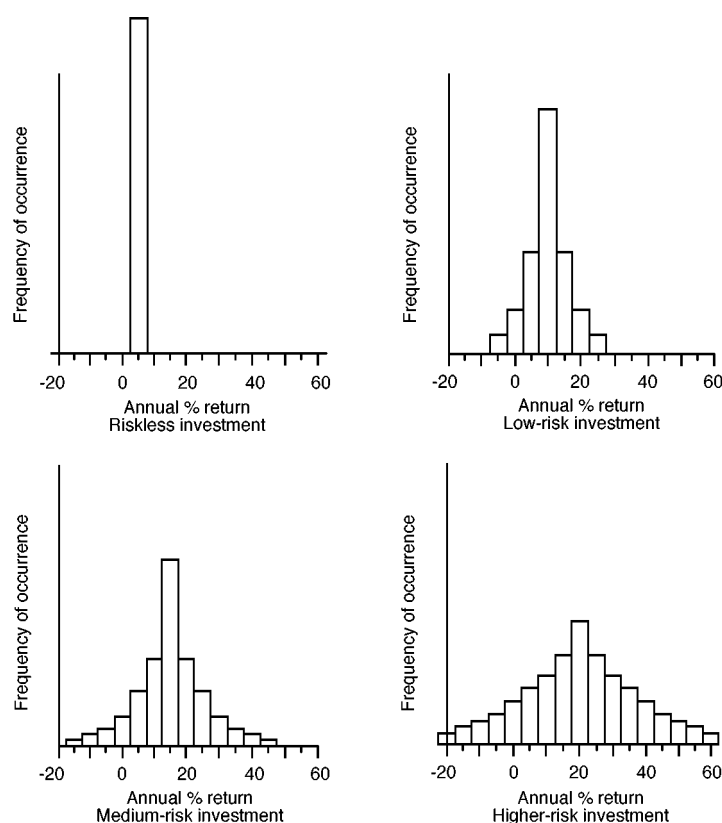
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There is an important difference, however, between investing in a savings account and investing in common stocks. The investor knows before committing any funds that the savings account will earn a return of 5%. The actual return will not differ from the expected return of 5%. Thus, savings accounts are considered a safe or risk-free security.

On the other hand, an investor who expects a return of 12% on Du Pont's common shares may be disappointed or pleasantly surprised. The actual return on Du Pont may be less than or greater than 12%, since (1) Du Pont may change its dividend and, more important, (2) the market price at the end of the period may differ from the anticipated price. Actual returns on common stock vary widely from year to year. An investor committing funds at the beginning of any period cannot be confident of receiving the average or expected return.

In general, an investment with actual returns that are not likely to depart from the expected or average return is considered a low-risk investment. One with quite volatile returns from year to year is said to be risky. Thus, risk can be viewed as variability in return (see **Figure A**).

Figure A Risk as Variability in Return



Risk Reduction through Diversification

Risky stocks can be combined in such a way that the combination of securities, called a portfolio of securities, is less risky than any one of the component individual stocks. Consider the example outlined in **Table A**. Suppose we have two firms located on an isolated Caribbean island. The chief industry on the island is tourism. Company A manufactures and sells suntan lotion. Its sales, earnings, and cash flows are highest during sunny years. Thus, its stock does well in sunny

years and poorly in rainy years. Company B manufactures and sells disposable umbrellas. Returns on its stock reflect its higher earnings in rainy years. In purchasing stock in either A or B, an investor is subject to considerable risk or variability in return. For instance, the investor's return on the stock of company B will vary from 33% to -9%, depending on weather conditions.

Table A Example of Risk Reduction through Diversification

	Weather Conditions	Return on Stock A = R_A
Company A: Suntan lotion manufacturer	Sunny year	33%
	Normal year	12
	Rainy year	-9
	Weather Conditions	Return on Stock B = R_B
Company B: Disposable umbrella manufacturer	Sunny year	-9%
	Normal year	12
	Rainy year	33
Returns on a Portfolio (R_p) Consisting of 50% Invested in Stock A and 50% in Stock B: (R_p) = .50 (R_A) + .50 (R_B)		
	Weather Conditions	Return on the Portfolio = R_p
Portfolio containing A and B	Sunny year	.50 (33%) + .50 (-9%) = 12%
	Normal year	.50 (12%) + .50 (12%) = 12%
	Rainy year	.50 (-9%) + .50 (33%) = 12%

Suppose, however, that instead of buying only one security the investor puts half of his or her funds in stock A and half in stock B. The possible returns on this portfolio of securities are calculated in **Table A**. If a recession occurs, a \$50 investment in stock A loses \$4.50, while \$50 invested in stock B returns \$16.50. The total return on \$100 invested in the portfolio is 12%:

$$\frac{-\$4.50 + \$16.50}{\$100}$$

Note that the return on this portfolio is 12% regardless of which weather condition prevails.

Combining these two risky securities yields a portfolio with a certain return. Since we are sure of earning 12% on the portfolio, it is a very low-risk investment comparable to a risk-free security such as a savings account. This example demonstrates risk reduction through diversification. By diversifying the investment over both firms, the investor creates a portfolio that is less risky than its two component stocks.

Total risk elimination is possible in this example because there is a perfect negative relation between the returns on stocks A and B. In practice, such a perfect relation is very rare. Most firms' securities tend to move together, and therefore complete elimination of risk is not possible. However, as long as there is some lack of parallelism in the returns of securities, diversification will always reduce risk. Since companies' fortunes and therefore their stocks' returns do not move completely in parallel, investment in a diversified portfolio composed of many securities is less risky than investment in a few individual stocks.

Systematic and Unsystematic Risk

Combining securities into portfolios reduces risk. When combined with other securities, a portion of a stock's variability in return is canceled by complementary variations in the returns of other securities. Some firms represented in the portfolio may experience unanticipated adverse conditions (e.g., a wildcat strike). However, this may well be offset by the unexpected good fortune of other firms in the portfolio. Nevertheless, since to some extent stock prices (and returns) tend to move in concert, not all variability can be eliminated through diversification. Even investors holding diversified portfolios are exposed to the risk inherent in the overall performance of the stock market (for instance, the stock market crash of October 1987). Thus, it is convenient to divide a security's total risk into that portion which is peculiar to a specific firm and can be diversified away (called unsystematic risk) and that portion which is market-related and nondiversifiable (called systematic risk):

$$\begin{array}{lcl} \text{Total risk} & = & \text{Unsystematic risk} \quad + \quad \text{Systematic risk} \\ & & \text{(diversifiable risk,} & & \text{(nondiversifiable risk,} \\ & & \text{firm-specific)} & & \text{market-related)} \end{array}$$

Figure B illustrates the reduction of total risk as securities are added to a portfolio. Unsystematic risk is virtually eliminated in portfolios of 30 or 40 securities drawn from industries that are not closely related. Since the remaining, systematic risk is market-related, diversified portfolios tend to move in tandem with the market. The popular market indices (the Dow-Jones Industrial Average, the S&P 500, and the New York Stock Exchange Index, for instance) are themselves diversified portfolios and tend to move in parallel. Thus, there is a close correspondence between swings in the returns of any diversified portfolio and in the returns on market indices such as the Dow. Examples of systematic and unsystematic risk factors are listed in **Table B**.

Figure B Elimination of Unsystematic Risk through Diversification

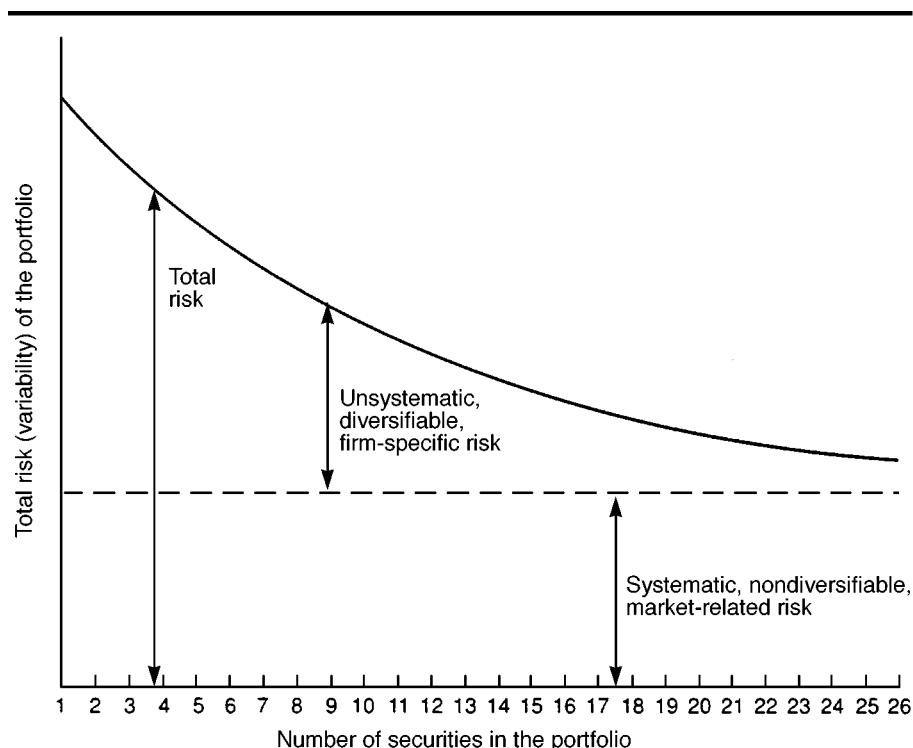


Table B Systematic and Unsystematic Risk Factors**Examples of Unsystematic Risk Factors**

A firm's technical wizard is killed in an auto accident.

A wildcat strike is declared.

A lower-cost foreign competitor unexpectedly enters a firm's product market.

Oil is discovered on a firm's property.

Examples of Systematic Risk Factors

Oil-producing countries institute a boycott.

Congress votes for a massive tax cut.

The Federal Reserve follows a restrictive monetary policy.

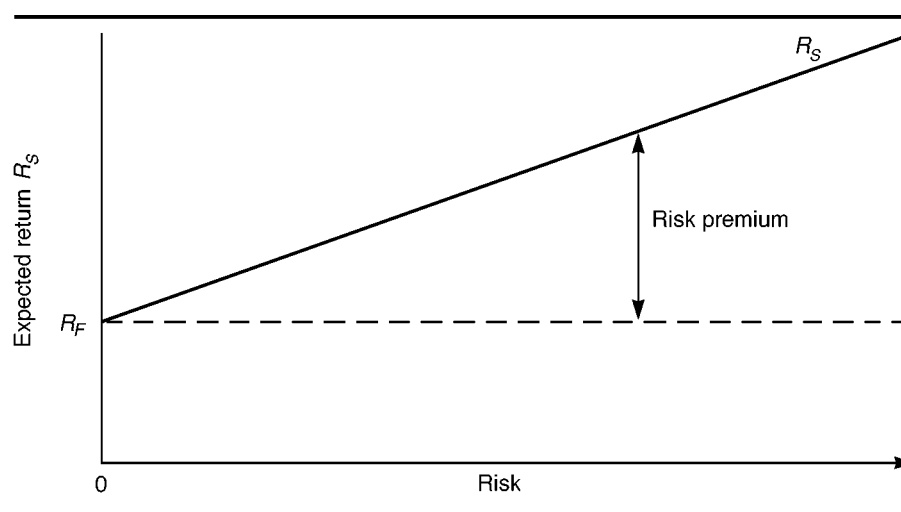
There is a precipitous rise in long-term interest rates.

Risk, Return, and Market Equilibrium

Investors are risk-averse, and they must be compensated for taking risk. Thus, risky securities are priced by the market to yield a higher expected return than low-risk securities. This extra reward, called the risk premium, is necessary to induce risk-averse investors to hold risky securities. In a market dominated by risk-averse investors, there must be a positive relation between risk and expected return to achieve equilibrium. The expected return on a risk-free security (such as a Treasury bill) is the risk-free rate. The expected return on risky securities can be thought of as this risk-free rate plus a premium for risk:

$$R_S = R_F + \text{Risk premium}$$

The market's risk/return trade-off is illustrated in **Figure C**.

Figure C Market's Risk/Expected Return Trade-Off in Equilibrium

The Capital Asset Pricing Model (CAPM)

The capital asset pricing model (CAPM) represents an idealized view of how the market prices securities and determines expected returns. It provides a measure of the risk premium and a method for estimating the market's risk/expected return curve.

In the CAPM, investors hold diversified portfolios to minimize risk. Since they hold portfolios consisting of many securities, events peculiar to specific firms (i.e., unsystematic risk) have a negligible impact on their overall return. Only a small fraction of an investor's funds is invested in each security. Furthermore, variations in returns from one security will, as likely as not, be canceled by complementary variations in the returns of other securities. Therefore, the only risk investors are sensitive to is systematic or market-related risk.

Since unsystematic risk can be eliminated simply by holding large portfolios, investors are not compensated for bearing unsystematic risk. Investors holding diversified portfolios are exposed only to systematic market-related risk. Therefore, the relevant risk in the market's risk/expected return trade-off is systematic risk, not total risk. The investor is rewarded with a higher expected return for bearing systematic, market-related risks. Only systematic risk is relevant in determining the premiums for bearing risk. Thus, the model predicts that a security's return is related to that portion of risk that cannot be eliminated by portfolio combination.

An individual investor who invests in only one stock is still exposed to both systematic and unsystematic risk. However, he or she is rewarded by a higher expected return only for the systematic risk he or she bears. There is no reward for bearing unsystematic risk, since it can be eliminated by adequate diversification.

The CAPM provides a convenient measure of systematic risk. This measure, called beta (β), gauges the tendency of a security's return to move in parallel with the overall market's return (e.g., the return on the S&P 500). A stock with a beta of 1 tends to rise and fall the same percentage as the market (i.e., the S&P 500 index). Thus, $\beta = 1$ indicates an average level of systematic risk. Stocks with $\beta > 1$ tend to rise and fall by a greater percentage than the market. They have a high level of systematic risk and are very sensitive to market changes. Similarly, stocks with $\beta < 1$ have a low level of systematic risk and are less sensitive to market swings.

These results determine the risk/expected return trade-off under the CAPM. In general,

$$R_S = R_F + \text{Risk premium}$$

If the CAPM correctly describes market behavior,

$$R_S = R_F + \beta_S (R_M - R_F)$$

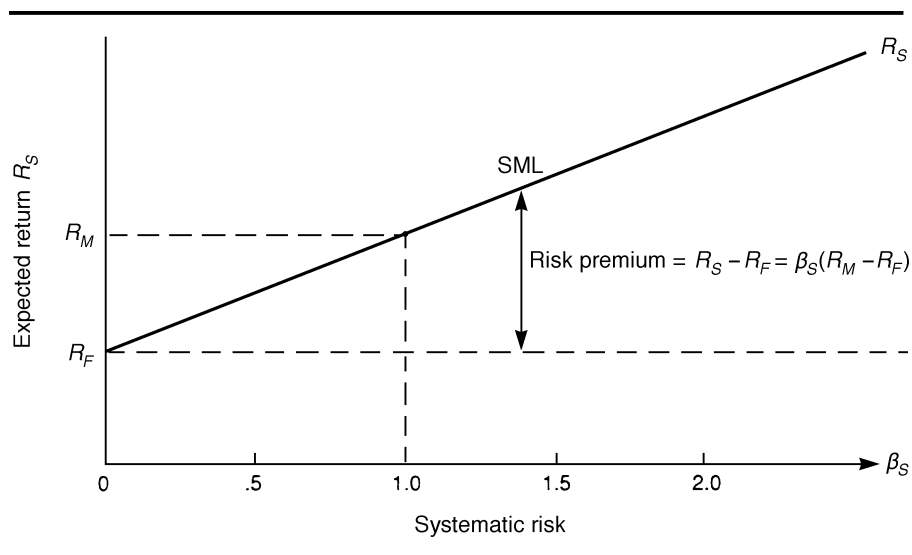
The expected return on a security (R_S) is equal to the risk-free rate plus a risk premium. With the CAPM, the risk premium is β multiplied by the return on the market (R_M) minus the risk-free rate. Alternatively, the relation can be expressed in terms of the risk premium (i.e., the return over and above the risk-free rate):

$$\begin{aligned} R_S - R_F &= \beta_S (R_M - R_F) \\ &= \text{Risk premium for security S} \end{aligned}$$

Thus, the risk premium on a stock (or portfolio or any security) varies directly with the level of systematic risk, β . This risk/expected return trade-off with the CAPM is called the Security Market Line (SML) and is illustrated graphically in **Figure D**.

One perhaps counterintuitive aspect of the determination of expected returns with the CAPM can be illustrated with a simple example. Consider a firm engaged in oil exploration. The return (denoted R_A) to the shareholders in such a firm is very variable. If oil is found, the return is very high. If no oil is discovered, shareholders lose their entire investment and the return is negative. The stock's total risk level is very high. However, much of the variability in return is generated by factors independent of the returns on other stocks (i.e., the return on the market). This risk is unique to the firm and is therefore unsystematic risk. Since the stock's return is not closely related to the return on the market as a whole, it contributes little to the variability of a diversified portfolio. Its unsystematic risk can be diversified away by holding large portfolios. Nevertheless, the costs of exploration and the price of oil are related to the general level of economic activity. As a result, the stock does contain some systematic, market-related risk. Most of its total risk is unsystematic risk, however, associated with the chances of finding oil.

Figure D Security Market Line: The Risk/Expected Return Trade-Off with CAPM



Although the firm's stock is very risky in terms of total risk, it has a low level of systematic risk. Its beta might be .8. The market will therefore price this stock to yield a relatively low expected return. From the viewpoint of investors holding large portfolios, it is a low-risk security. Its expected return is denoted R_A in **Figure E**. Note that the return on this stock (R_A) is less than the return on the average stock in the market (R_M).

In contrast, consider a firm that manufactures computers. As a large stable firm, its total variability in return might be less than that of the oil exploration firm. However, its sales, earnings, and therefore stock returns are closely related to changes in overall economic activity. The return on its stock is very sensitive to changes in the return on the market as a whole. Therefore, its risk cannot be eliminated by diversification. When combined with other securities in a diversified portfolio, changes in its return tend to reinforce swings in the returns of the other securities. It has a relatively high level of systematic risk and a beta of perhaps 1.2. Viewed as an individual security, it appears less risky (in terms of total risk) than the oil exploration firm. Nevertheless, because of its high level of nondiversifiable risk, the market considers it the riskier security. Therefore, it is priced to yield a high expected return. Its return is labeled R_B in **Figure E**. Such counterintuitive examples are rare, however. Most firms with high total risk also have high betas (and vice versa).

In summary, if the CAPM correctly describes market behavior, the relevant measure of a security's risk is its market-related or systematic risk (measured by beta). If a security's return has a

strong positive relation with the return on the market (i.e., has a high β), it will be priced to yield a high expected return (and vice versa). Since unsystematic risk can be easily eliminated through diversification, it does not increase a security's expected return. The market cares only about systematic risk. These results are summarized in **Table C**.

Figure E Example of Determining Expected Returns with the CAPM

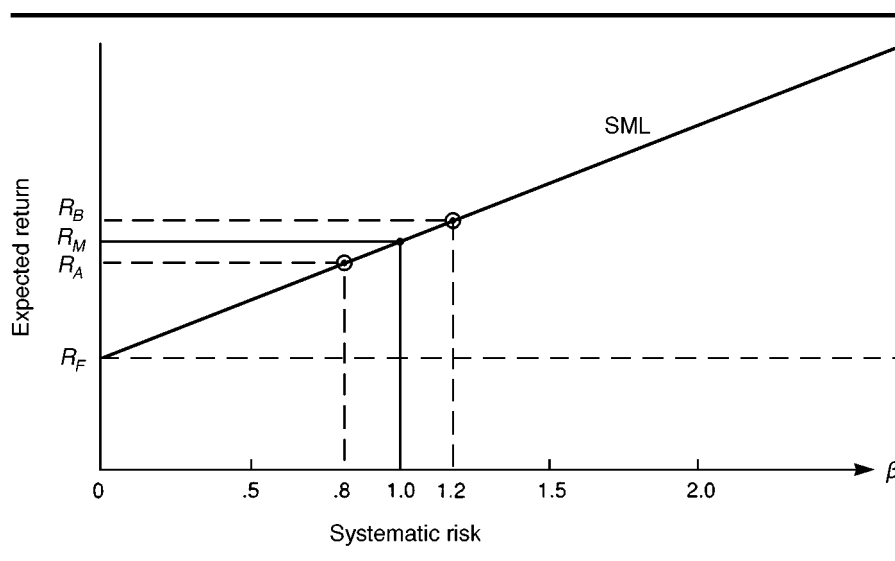


Table C Summary of the Determination of Expected Returns with CAPM

1. Total risk is defined as variability in return.
2. The investor can reduce risk by holding a diversified portfolio.
3. The total risk of a security can be divided into unsystematic and systematic risk.
 - a. Risk that can be eliminated through diversification is called unsystematic risk. It is associated with events unique to the firm and independent of other firms.
 - b. The risk remaining in a diversified portfolio is called systematic risk. It is associated with the movement of other securities and the market as a whole.
4. If CAPM correctly describes market behavior, investors hold diversified portfolios to minimize risk.
5. Since investors hold diversified portfolios with the CAPM, they are exposed only to systematic risk. In such a market, investors are rewarded by a higher expected return only for bearing systematic, market-related risk. There is no reward associated with unsystematic risk because it can be eliminated through diversification. Thus, relevant risk is systematic or market-related risk, and it is measured by beta.
6. The risk/expected return trade-off with the CAPM is called the Security Market Line (SML). Securities are priced such that:

$$R_S = R_F + \text{Risk premium, or } R_S = R_F + b_S (R_M - R_F)$$

Thus, the SML gives us an estimate of the expected return on any security, R_s .

Application of the CAPM to Corporate Finance: Estimating the Cost of Equity Capital

The CAPM provides insight into the market's pricing of securities and the determination of expected returns. It has clear applications in investment management and in corporate finance. The cost of equity capital, k_E , is the expected (or required) return on a firm's common stock. The firm must be expected to earn k_E on the equity-financed portion of investments to keep the price of its stock from falling. If the firm cannot expect to earn at least k_E , funds should be returned to the shareholders, who can earn k_E on marketable securities of the same risk level. Since k_E involves the market's expectations, it is difficult to measure. The CAPM can be used by financial managers to obtain an estimate of k_E .

The CAPM provides a conceptual framework for determining the expected return on common stocks, and it can be used to estimate firms' cost of capital. If the CAPM correctly describes market behavior, the market's expected return on a common stock is given by the Security Market Line (SML):

$$R_s = R_F + \beta_s (R_M - R_F)$$

The expected return on a firm's stock is, by definition, its cost of equity capital. Therefore, in terms of cost of capital, the SML is

$$k_E = R_F + \beta_s (k_M - R_F)$$

where

$k_E = R_s =$ firm's cost of equity capital

$k_M = R_M =$ cost of equity for the market as a whole (or for an average firm in the market)

$\beta_s =$ beta of the firm's stock

Thus, to estimate k_E we need estimates of R_F , the risk-free rate; $k_M = R_M$, the expected return on the market as a whole; and β_s , the level of systematic risk associated with the firm's stock.

R_F can be estimated as the average or expected rate of return on Treasury bills in the future. In recent years, this rate has ranged between 5% and 10%. A reasonable estimate might be 9% per year.

The market risk premium is the difference between the return on the market, k_M , and the risk-free rate, R_F . The expected risk premium in the future is difficult to estimate. A common approach is to assume investors expect returns in the future to be about the same as returns in the past. The average annual market risk premium (equities versus Treasury bills) was 8.4% in the period 1926-1990.¹

The stock's beta, β_s , can be estimated by linear regression.² Betas are also available from many brokerage firms and investment advisory services. Furthermore, one can get an intuitive

¹Stocks, Bonds, Bills, and Inflation—1991 Yearbook: Market Results for 1926-1990 (Chicago: Ibbotson Associates, 1991).

²The estimated regression equation is $R_s - R_F = \alpha + \beta_s(R_M - R_F) + e$. Given past values of R_F , R_s , and R_M , the regression yields estimates of α (which should be zero) and the stock's beta, β_s .

estimate simply by observing the stock's reaction to swings in the market as a whole. Finally, a rough guess at beta can be made by noting the tendency of the firm's earnings and cash flows to move in parallel with the earnings and cash flows of other firms in the economy.

Betas for selected firms in four industries are presented in **Table D**. Despite relatively high degrees of operating and financial leverage, electric utilities have very stable earnings streams. Swings in the earnings and stock returns of utilities are modest relative to swings in the earnings and returns of most firms in the economy. Therefore, electric utilities have a low level of systematic risk and low betas.

Table D Betas for Selected Firms in Four Industries

Electric Utilities		Airlines		Computer Hardware		Computer Software	
Company	β	Company	β	Company	β	Company	β
American Electric Power	.56	AMR Corp.	1.20	Apple Computer	1.93	Abode Systems	2.11
Baltimore Gas & Electric	.64	Delta	.98	Compaq	1.25	Aldus Corp.	1.85
Commonwealth Edison	.55	Northwest	.91	Digital Equip.	1.22	Ashton-Tate	1.99
Consolidated Edison	.60	United	1.18	Hewlett-Packard	1.31	Borland International	1.86
Duke Power	.56	USAir Group	1.16	IBM	1.03	Computer Assoc.	1.35
Niagara Mohawk	.61			Tandy	1.28	Ingres Corp.	1.49
Ohio Edison	.53					Lotus Development	1.86
Pacific Gas & Electric	.57					Microsoft	1.69
Philadelphia Electric	.53					Novell Inc.	1.98

At the other extreme, airline revenues are closely tied to passenger miles, which are in turn very sensitive to changes in economic activity. This basic variability in revenues is amplified by high operating and financial leverage. The result is earnings and returns that show wide variations relative to swings in the earnings and returns of most firms. Thus, airlines have high betas.

Estimates of the cost of equity capital for four firms are presented in **Table E**. Plugging the assumed values of R_F , k_M , and β into the SML generates estimates of k_E . As expected, the low-risk utility has an estimated cost of equity below that of the other three firms.

The assumed value of k_M represents a major potential source of error in these estimates. High and low estimates of k_M can be used to generate a reasonable range of estimates of k_E . The estimation of β also introduces error into the estimate of k_E .

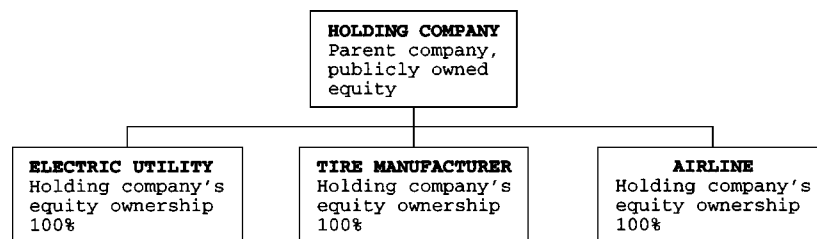
The CAPM and Risk-Adjusted Discount Rates

The CAPM provides a conceptual framework for determining the k_E appropriate for a subsidiary's capital budgeting decisions. Assume that one holding company described in **Figure F** has no debt outstanding. The parent company owns all the equity in its subsidiaries, and the holding company's stock is publicly traded.³ Such a firm can be viewed as a portfolio of assets. Its stock's beta is a weighted average of the betas associated with the riskiness of each subsidiary industry.

³The cost of equity is appropriate to evaluate capital investment only when the firm is all equity financed. The note "Leveraged Betas and the Cost of Equity" explains how to estimate the cost of capital for firms that are financed with debt.

Table E Examples of Estimating the Cost of Equity Capital Using the CAPM

Assumptions			
$R_F = .09 = \text{risk-free rate}$			
$R_M - R_F = .08$			
SML			
$k_E = R_F + \beta(k_M - R_F)$			
$= .09 + \beta(.08)$			
Commonwealth Edison	United Airlines	IBM	Lotus
$\beta_{\text{Commonwealth}} = .55$	$\beta_{\text{United}} = 1.18$	$\beta_{\text{IBM}} = 1.03$	$\beta_{\text{Lotus}} = 1.86$
$k_E = .09 + .55(.08)$	$k_E = .09 + 1.18(.08)$	$k_E = .09 + 1.03(.08)$	$k_E = .09 + 1.86(.08)$
$= .13$	$= .18$	$= .17$	$= .24$

Figure F Corporate Structure of a Holding Company with Three Subsidiaries

Suppose that the parent company's beta is 1. However, the appropriate cost of equity capital for capital budgeting purposes is not the k_E derived from the beta of the holding company's stock. The cost of equity capital used to evaluate investment proposals for a subsidiary should reflect the risk associated with the industry in which that subsidiary operates. Thus, while the holding company's beta of 1 yields a k_E of 15%, investments in the utility subsidiary should be evaluated using a lower k_E , since the utility industry is less risky than the other subsidiary industries. Therefore, the market's expected (or required) return is lower for investments in the utility subsidiary. Since the airline industry is risky, a higher k_E should be used in capital budgeting for an airline subsidiary.

Application of the firm's overall k_E to the individual subsidiaries would result in poor decisions. Good projects in the utility subsidiary would be rejected while poor projects in the airline subsidiary would be accepted. When the cost of equity capital used in a subsidiary's capital budgeting decisions reflects the risk associated with that subsidiary's line of business, this ensures that project returns are measured against the returns shareholders would expect to receive on alternative investments of corresponding risk.

How can we estimate the beta appropriate for a subsidiary? An obvious approach is to use the beta for similar independent firms operating in the same industry. The resulting estimates of k_E reflect the risk level of the industry and are therefore appropriate for investment decisions concerning a subsidiary operating in the same industry. If there are no independent firms in the industry, an intuitive estimate of beta can be made. This estimate would reflect the degree to which the subsidiary's earnings and cash flows tend to move in concert with other firms' earnings and cash flows.

Conclusion and Caveats

The CAPM is widely applied in investment management and corporate financial management. Although some of the model's assumptions are clearly unrealistic, empirical tests demonstrate that there is a strong relation between returns and risk as measured by beta. However, the nature and stability of the relations predicted by the SML are not fully supported by these tests. Furthermore, application of the CAPM requires estimating $k_M - R_F$, the market risk premium, and R_F , the risk-free rate. The estimates of beta are also subject to error. Thus, the CAPM should not be relied upon as the sole answer to cost of capital determination.

Nevertheless, the model has much to say about the way returns are determined in the securities market. The cost of equity capital is inherently difficult to measure. The shortcomings of the CAPM appear less severe than those of alternative methods of estimating the cost of equity capital (for instance, the dividend growth model). Though imperfect, the CAPM represents an important approach to this difficult task. Using the CAPM in conjunction with more traditional approaches, corporate financial managers can develop realistic, useful estimates of the cost of equity capital.